NUMERICAL DIFFERENTIATION

US05CPHY25

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Numerical Differentiation

Theory: The operation used in the calculus to obtaining the differentiation coefficient is known the differentiation.

For example, if y= f'(x) = x^n then the differentiation co efficient: $\frac{dy}{dx} = nx^{n-1}$

Where, $\frac{dy}{dx}$ is known as the first order derivative and is denoted by y',

similarly, $\frac{d^2y}{dx^2}$ is the second order derivative and is denoted by y'' and so on…

To determine the derivative of any function f(x) at a given point x=a, the formula can be used to get an approximate answer, suppose f(x) is any function then the derivative can be calculated by,

$f'(x) = \frac{1}{h}\,[f(x+h) - f(x)]_{x=a} \pm e$, Where, h - is step length and e - is an error.

Error can be calculated by,

$e = \frac{h}{2}[f''(x)]$, where, $f''(x) = \frac{1}{h^2}\,[f(x+h) - 2f(x) + f(x-h)]$

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Procedure:

1. Make tables using given equation for various methods.
2. Compute the value of given equations, by:
   - (i), Analytical method,
   - (ii), Numerical method, and
   - (iii), Graphical method.
3. Show and compare results and find out error.

Example:

Find the value of $f'(x)$ for a given function y = $f(x) = x^2$ at $x = 2$ and h = 0.1.

(a). Analytical method:

Here, $y = f(x) = x^2$ is given,

So, $y = f(x) = x^2$

then $y' = f'(x) = \frac{dy}{dx} = 2\,x^{2-1} = 2\,x$

But $x = 2$,

$\therefore\ y' = f'(x) = \frac{dy}{dx} = 2 \times 2 = 4$

(b). Numerical method:

Here, y = $f(x) = x^2$ and x =2, h=0.1 are given,

Now, we know that,

$$y' = f'(x) = \frac{1}{h}\,[f(x+h) - f(x)]_{x=a} \pm e$$

1. $f(x) = x^2 = 2^2 = 4$
2. $f(x+h) = f$ (2+0.1) = f(2.1)

   But, $f(x) = x^2$, therefore, f (2.1) = $(2.1)^2 = 4.41$
3. To find error, first of all we should find the value of $f''(x)$.

   (i) $f(x) = x^2 = 2^2 = 4$,

   (ii) $f(x+h) = 4.41$and

   (iii) $f(x) - h$ = (2-0.1) = (1.9),

but $f(x) = x^2 = (1.9)^2$ =3.61.

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Now, $f''(x) = \frac{1}{h^2}\,[f(x+h) - 2f(x) + f(x-h)]$

Substituting above values,

$$f''(x) = \frac{1}{h^2}\,[f(x+h) - 2f(x) + f(x-h)]$$

$$f''(x) = \frac{1}{(.1)^2}\,[f(4.41) - 2f(4) + f(3.61)] = 2$$

Now, error $e = \frac{h}{2}[f''(x)] = -\frac{0.1}{2}[2]$ = - 0.1

Let, $y' = f'(x) = \frac{1}{h}\,[f(x+h) - f(x)]_{x=a} \pm e$

$$y' = f'(x) \approx \frac{1}{o.1}[4.41 - 4]_{x=2} - 0.1 = \frac{0.41}{o.1} = 4.1 \approx 4$$

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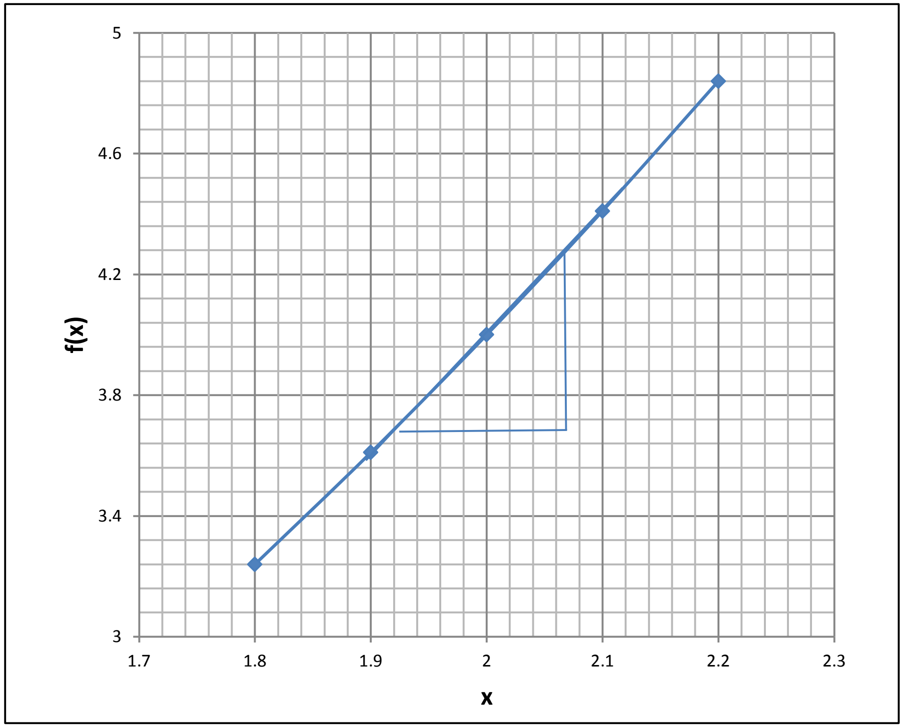
(c). Graphical method: Make a table for given function andPlot a graph between x and f(x) and find out the value of slope you will get the value of a function.

Table

Sl. No.	1	2	3	4	5
x	1.8	1.9	2.0	2.1	2.2
y = f(x)	3.24	3.61	4.0	4.41	4.84



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Slope of the tangential line: $\mathbf{m} = \frac{ab}{bc}$ = ___

RESULT

No.	Function $f(x)$	Value of differentiation by		
		Analytical Method	Numerical Method	Graphical Method
1.				
2.				
3.				

EXERCISE

$y = x^3$	$y = (x+x^2)^2$	$y = 2x^3$	$y = e^x$	$y = \log_{10} x$	$y = (x^2+1)^3$

At $x = 2$ or any value is given